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FROM: J. J. Schoch

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BELLCOMM, INC.1100 Seventeenth Street, N.W. Washington, D. C. 20036

SUBJECT: Probe Deployment to the Planet
Mars from an Elliptical Parking
Orbit, Case 720

DATE: July 19, 1968

FROM: J. J. Schoch

MEMORANDUM FOR FILE

INTRODUCTION

Planetary probe injection from hyperbolic flyby trajectories was investigated in two previous memoranda^{1,2}. For a Mars stopover mission it will be assumed that the spacecraft orbits the planet on an elliptical orbit. Probe deployment from this elliptical parking orbit is investigated on the basis of two criteria: ΔV expenditure and range coverage on the planetary surface.

ELLIPTICAL ORBITS AND ENTRY CONDITIONS

The elliptical orbit around Mars that was considered is characterized by:

Period of revolution 24 hours

Periapsis Radius 100 N.M.

(This corresponds to an apoapsis radius of about eleven planetary radii.)

Entry conditions were as follows:

Entry altitude: 700,000 Ft.

Entry angle: -13.5 and -15.5 Degrees

The higher flight path angle would result in a 5 g load factor for an Apollo type entry vehicle with $L/D = .36$, while the shallower angle was chosen as a limit below which the range sensitivity becomes excessive.

ANALYSIS

It is required to find a conic section that departs from the parking ellipse and enters the atmosphere at the conditions given above. Reference is now made to Figure 1, which shows the parking ellipse and one such transfer ellipse. It is assumed that the spacecraft travels on the parking ellipse in counter-clockwise direction. At point P* having an anomaly θ^* a probe is deployed

with a velocity increment ΔV such that it follows the transfer ellipse shown and enters the atmosphere with the required entry conditions at point P_2 with anomaly θ_2 .

A computer program named ELPAR was developed that determines transfer conics departing from specified anomalies along the parking orbit and all entering the atmosphere at one entry anomaly and entry angle. The derivation of the equations used in the program is given in the Appendix.

Of particular interest among the many possible transfer conics is that combination of deployment and entry anomaly giving the smallest deployment ΔV requirement. Graph 1A shows a plot of deployment ΔV vs. the anomaly of the deployment point with entry anomaly as parameter, for an entry angle of -15.5 degrees. Graph 1B shows the corresponding plot for an entry angle of -13.5 degrees. From these graphs, it may be seen that for most of the anomalies, the deployment ΔV lies above 1000 ft/sec. The parameter curves for entry anomalies between about 310 and 335 degrees have some points between 100 and 1,000 ft/sec and only for a very narrow range of about 5 degrees in entry anomalies does the ΔV expenditure go below 100 ft/sec. On Graph 2, the min. ΔV of each of the parameter curves of Graph 1 has been plotted vs. the entry point anomaly. This graph shows even more distinctly the very narrow region of entry anomalies for which the deployment ΔV goes below 100 ft/sec: between 328-332 degrees for a -13.5 degrees entry angle or 324-328 degrees for an entry angle of -15.5 degrees.

A small change in periapsis altitude does not change these results significantly. Increasing it from 100 n.m. to 130 n.m. for the -15.5 degree entry increased the minimum ΔV requirement by about 20%.

An increase in the period of revolution to 48 hours, corresponding to an apoapsis radius of almost 18 planetary radii, gives a minimum ΔV of approximately one half of that of the standard case. A reduction to 12 hours which corresponds to an apoapsis radius of a little over six planetary radii increases the minimum deployment ΔV to about 85 ft/sec.

In all the previous considerations, it has been assumed that, upon approaching the planet on a hyperbola, the transfer from the hyperbola to the parking ellipse is executed at periapsis, which provides the most economical transfer. The orientation of the parking ellipse with respect to the hyperbola may be changed by transferring into the ellipse before or after periapsis passage, or transferring from periapsis of the hyperbola to a non-periapsis point of the ellipse. This is essentially the inverse problem as described by H. S. London in Reference 3. A computer program INVSMP, that utilizes the method described therein, was used to

obtain the results given in Graph 3. Data were run for $V_{\infty} = 16,000$ and $22,000$ ft/sec, and for changes in orientation, $\Delta\psi$ of the major axis of the ellipse of ± 50 degrees.

If the mission is a roundtrip with a manned spacecraft, this maneuver implies that the probe (which may be a manned landing capsule) would orbit on and enter from an ellipse having a different orientation than that of the parent spacecraft since the latter ellipse will have an orientation that approximately minimizes the total ΔV for Mars capture and escape. It may even be necessary to separate the probe from the parent spacecraft a few days ahead of time in order to approach the planet on a hyperbola with a different periapee altitude.

Comparison of Graph 2 and Graph 3 shows that early or late injection is about twice as effective in increasing the landing coverage as changing the entry point anomaly on the parking ellipse. According to Graph 2, the anomaly of the entry point may be changed about 18 degrees in one direction and 9 degrees in the other (vs. the minimum ΔV entry point) for an expenditure of $1,000$ ft/sec on the nominal ellipse. If the same ΔV expenditure is applied to the earlier or later transfer from the approach hyperbola to the parking ellipse, the latter may, according to Graph 3, be rotated about 30 degrees on either side. A minimum ΔV entry may then be effected from this ellipse.

Since the slope of Graphs 2 and 3 are not uniform, it will occasionally pay to combine both methods for changing the anomaly of the entry point. For a ΔV expenditure below 1000 ft/sec, the slope on Graph 3 is smaller than the one on Graph 2, and the whole maneuver should be made by ellipse orientation. For larger ΔV 's, it will pay to have a portion of it expended by using an off-optimum entry anomaly (Graph 2). The ratio of the two contributions will have to be selected according to the slopes of the respective curves.

When entering the Martian atmosphere with an entry angle of -15.5 degrees, it takes a probe with $L/D = 0.36$ about 700 n.m. or 22 degrees to reach the surface (see Figure 2.). This distance becomes twice as large, i.e., 44° when entering the atmosphere at an angle of -13.5° . Furthermore, entry takes place 5° further downrange. As a consequence, an arc of 27° may be covered on the planetary surface by changing the entry angle between -13.5 to -15.5 . This adds directly to the range variation which can be obtained by selection of the capture and/or deorbit maneuver.

For comparison, runs were made from a circular orbit. The altitude of the orbit was $700,000$ ft (115 n.m.) and the entry angle -6.0 degrees which provides about the same range sensitivity as the -13.5 degrees entry for the ellipse. The minimum ΔV

requirement is 1180 ft/sec. Due to symmetry, any entry anomaly within the orbit plane may be reached on the planet. The variation of ΔV vs. the anomaly of the deployment point for an entry point anomaly of 0 degrees is shown on Graph 4. The deployment velocity increment remains fairly constant over a wide range of deployment anomalies. A further advantage of the circular parking orbit is of course the fact that, during a twenty-four hour period, the circular orbit having a period of 1.8 hours provides about 13 launch opportunities while only one launch opportunity is provided from an elliptical parking orbit. The escape or capture ΔV for a circular orbit is of course considerably larger than for an elliptical orbit.

CONCLUSIONS

Probes to Mars may be deployed from an elliptical parking orbit having a periapsis altitude of 100 n.m. for less than 100 ft/sec. This result does not change significantly by increasing or decreasing the period of revolution by a factor of two or increasing the periapsis altitude by 30%.

The range of entry anomalies for which the deployment ΔV goes below 100 ft/sec is very limited: 328-332 degrees for an entry angle of -13.5 degrees and 324-328 degrees for an entry angle of -15.5 degrees. The probe has to be deployed close to apoapsis in order to obtain these entry anomalies.

Changing the orientation of the probe parking ellipse with respect to the flyby hyperbola (Graph 3) is about twice as effective in increasing the range of entry anomalies as compared with deployment from non-optimum points on the nominal ellipse. For 1000 ft/sec the orientation of the probe parking ellipse can be changed by about ± 30 degrees.

Variation of the entry angle from 13.5 degrees to 15.5 degrees provides an additional range in landing anomalies of 27 degrees.

Almost 1200 ft/sec is required for deployment from a 115 n.m. altitude circular orbit. For this ΔV , however, any anomaly on the circumference may be reached.

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Attachments

Appendix

Figures 1 and 2

Graphs 1, 2, 3, and 4

References

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APPENDIX

DERIVATION OF EQUATIONS DEFINING THE TRANSFER TRAJECTORY

Let the parking ellipse be defined by its periapsis altitude h_p (n.m.) and its period of revolution P (sec) around a planet having a gravitational constant μ (ft^3/sec^2) and a radius R (ft).

From these elements, the semi-major axis is:

$$a = \sqrt[3]{\mu \left(\frac{P}{2\pi} \right)^2}$$

and the eccentricity:

$$e = 1 - \frac{R + h_p}{a}$$

Let the transfer ellipse be defined by the entry radius r_2 (ft), the entry flight path angle γ_2 (deg), and the true anomaly of the entry point with respect to the parking ellipse θ_2 . Let α be the angle between the directions of the periapsides of the ellipses and ϕ_2 the anomaly of the entry point with respect to the transfer ellipse. Then:

$$\phi_2 = \theta_2 + \alpha$$

Similarly for the deployment point P^* :

$$\phi^* = \theta^* + \alpha$$

The equation of the transfer conic with p , semilatus rectum, may be written as:

$$\cos \phi^* = \frac{1}{e_t} \left(\frac{p_t}{r^*} - 1 \right) \quad (1)$$

Appendix (Continued)

for the deployment point and as:

$$p_t = r_2 (e_t \cos \phi_2 + 1) \quad (2)$$

for the entry point.

Substituting (2) into (1) and collecting terms:

$$\cos \phi^* = \frac{r_2}{r^*} \cos \phi_2 + \frac{1}{e_t} \left(\frac{r_2}{r^*} - 1 \right) \quad (3)$$

Furthermore, the relation

$$\sin \phi_2 = \frac{p_t \tan \gamma_2}{e_t r_2} \quad (4)$$

may be used and substituting (2) for p_t in (4) above

$$\sin \phi_2 = \frac{(e_t \cos \phi_2 + 1) \tan \gamma_2}{e_t}$$

and solving for e_t

$$e_t = \frac{\tan \gamma_2}{\sin \phi_2 - \cos \phi_2 \tan \gamma_2} \quad (5)$$

Substituting (5) above for e_t in (3).

$$\cos \phi^* = \left(\frac{r_2}{r^*} - 1 \right) \frac{\sin \phi_2}{\tan \gamma_2} + \cos \phi_2$$

and with:

$$\sin \phi_2 = \sin (\theta_2 + \alpha) = \sin \theta_2 \cos \alpha + \cos \theta_2 \sin \alpha$$

$$\cos \phi_2 = \cos (\theta_2 + \alpha) = \cos \theta_2 \cos \alpha - \sin \theta_2 \sin \alpha$$

$$\cos \phi^* = \cos (\theta^* + \alpha) = \cos \theta^* \cos \alpha - \sin \theta^* \sin \alpha$$

Appendix (Continued)

and furthermore setting

$$\left(\frac{r_2}{r^*} - 1\right) \frac{1}{\tan \gamma_2} = c$$

and solving for α

$$\tan \alpha = \frac{\cos \theta^* - \cos \theta_2 - c \sin \theta_2}{\sin \theta^* - \sin \theta_2 + c \cos \theta_2}$$

This determines the inclination between the two conics and permits to determine the elements of the transfer conic.

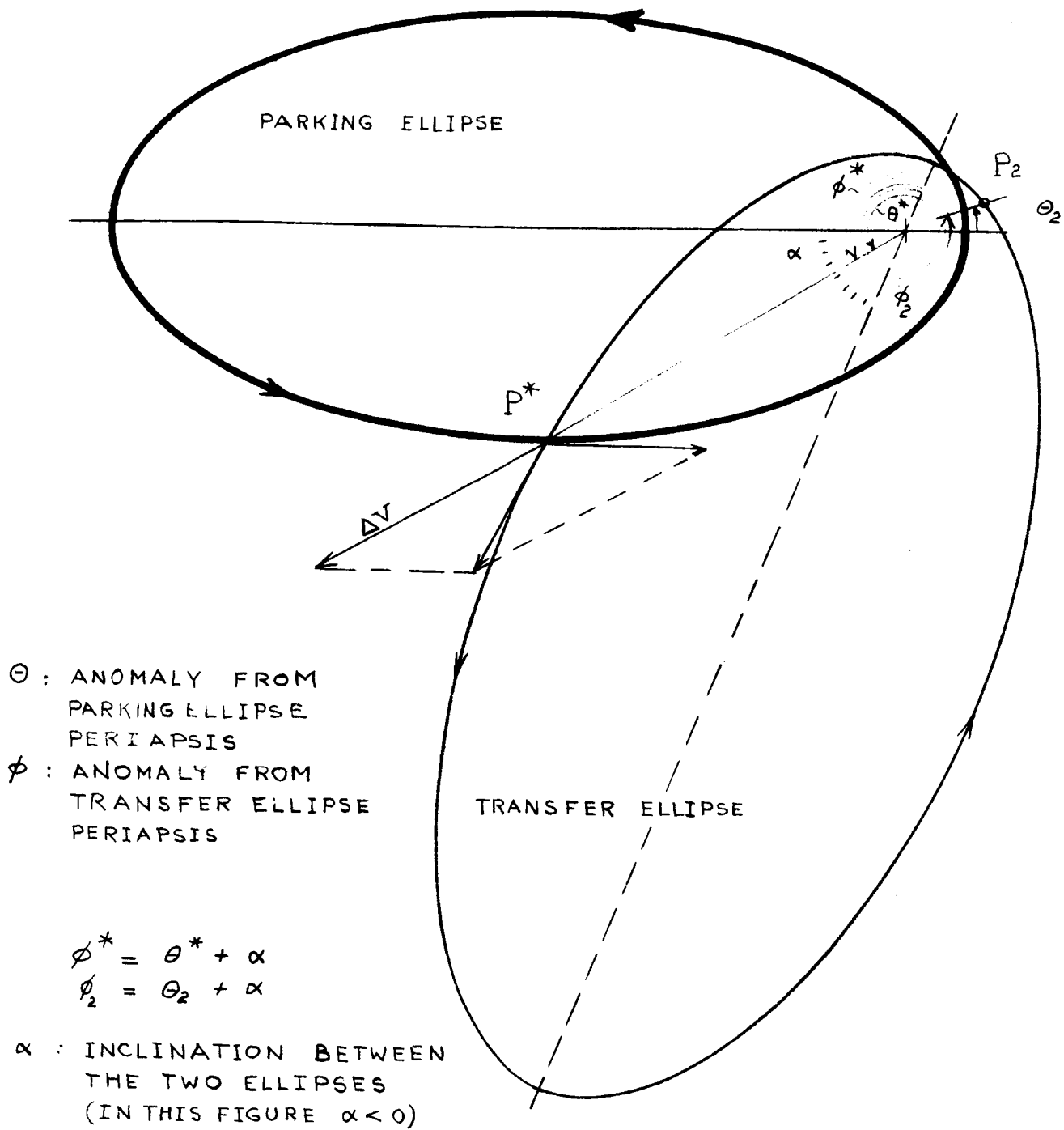
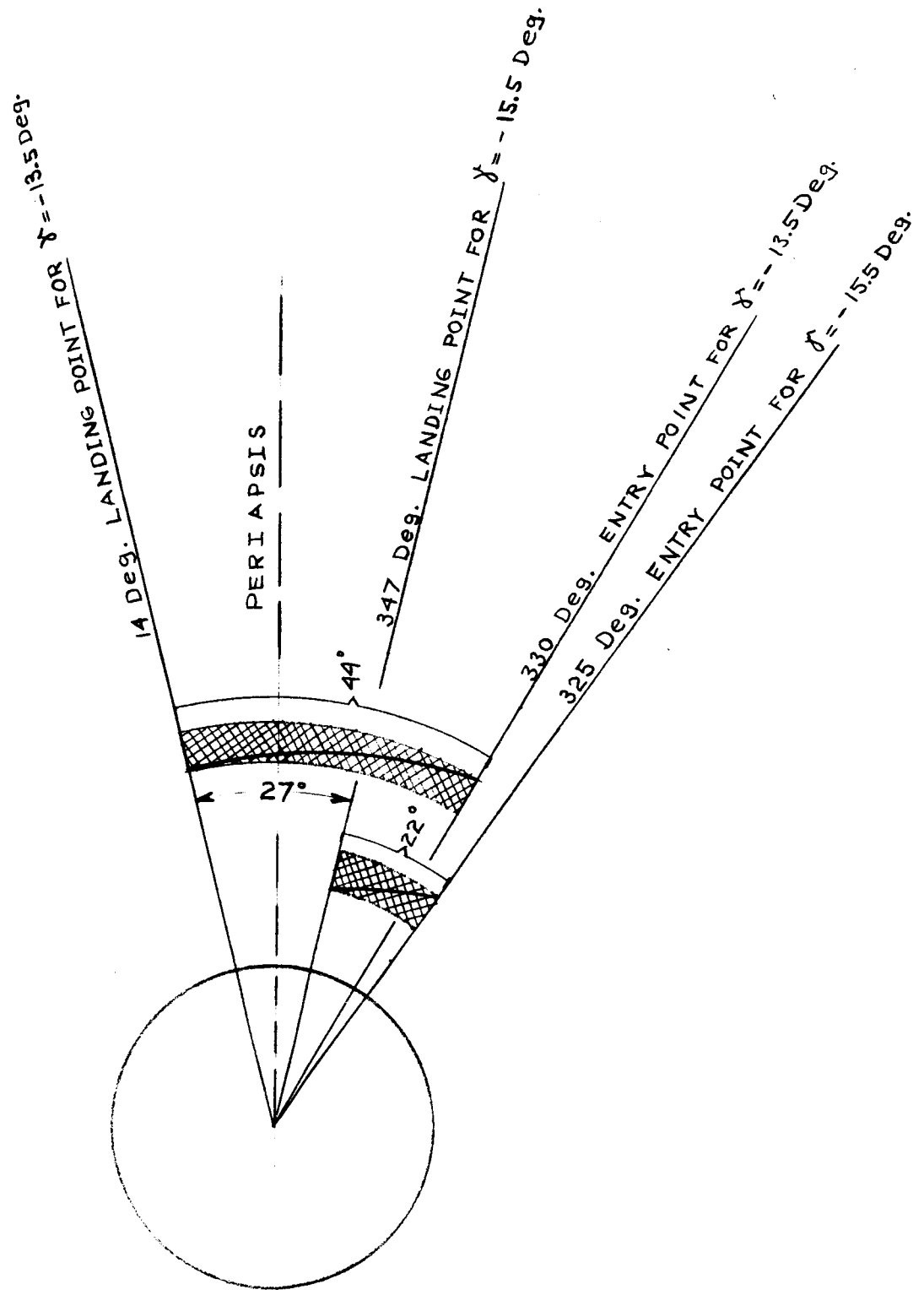
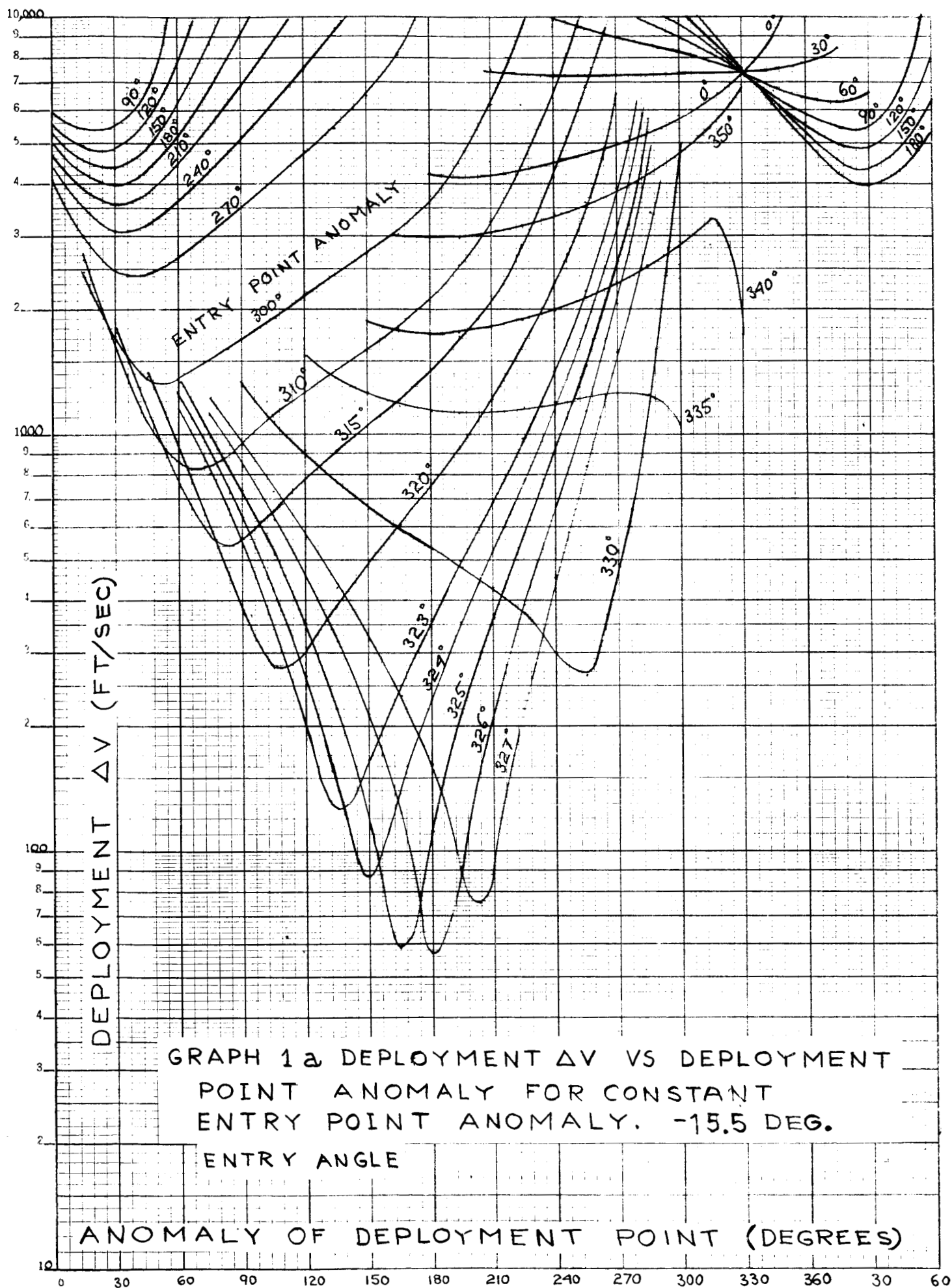
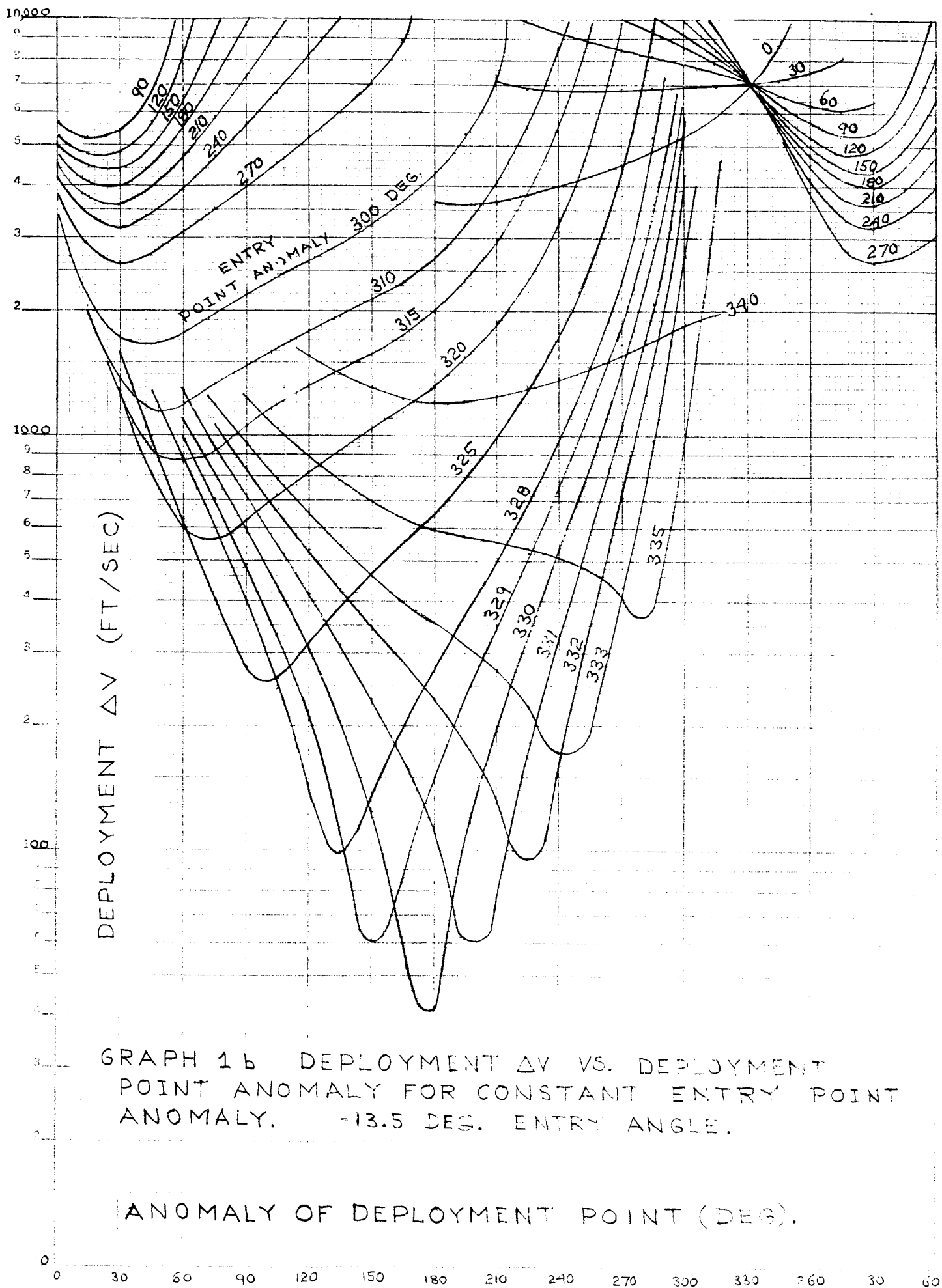


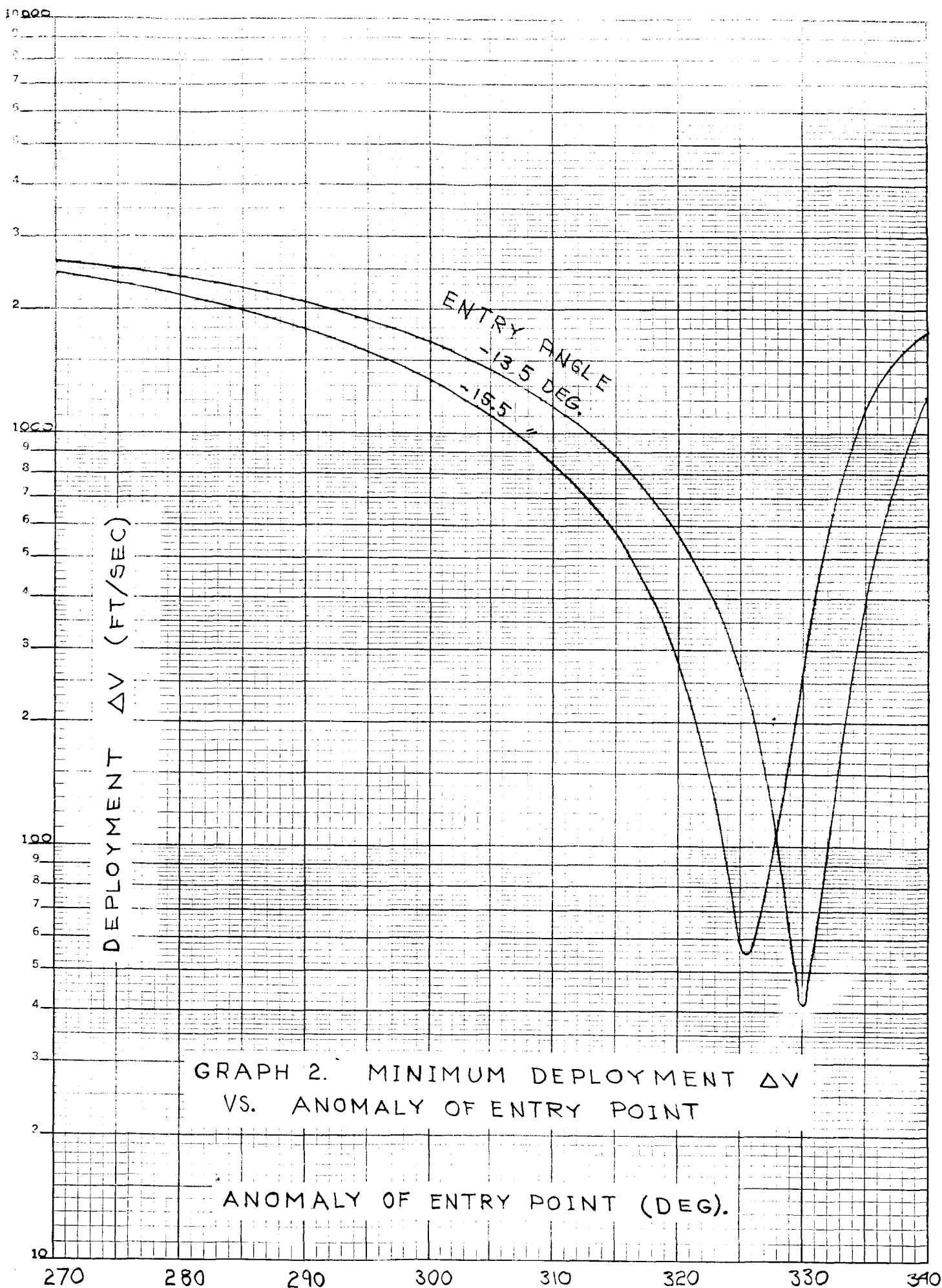
FIGURE 1
PARKING & TRANSFER ELLIPSE.

FIGURE 2 LOCATION OF ENTRY- AND LANDING POINTS.

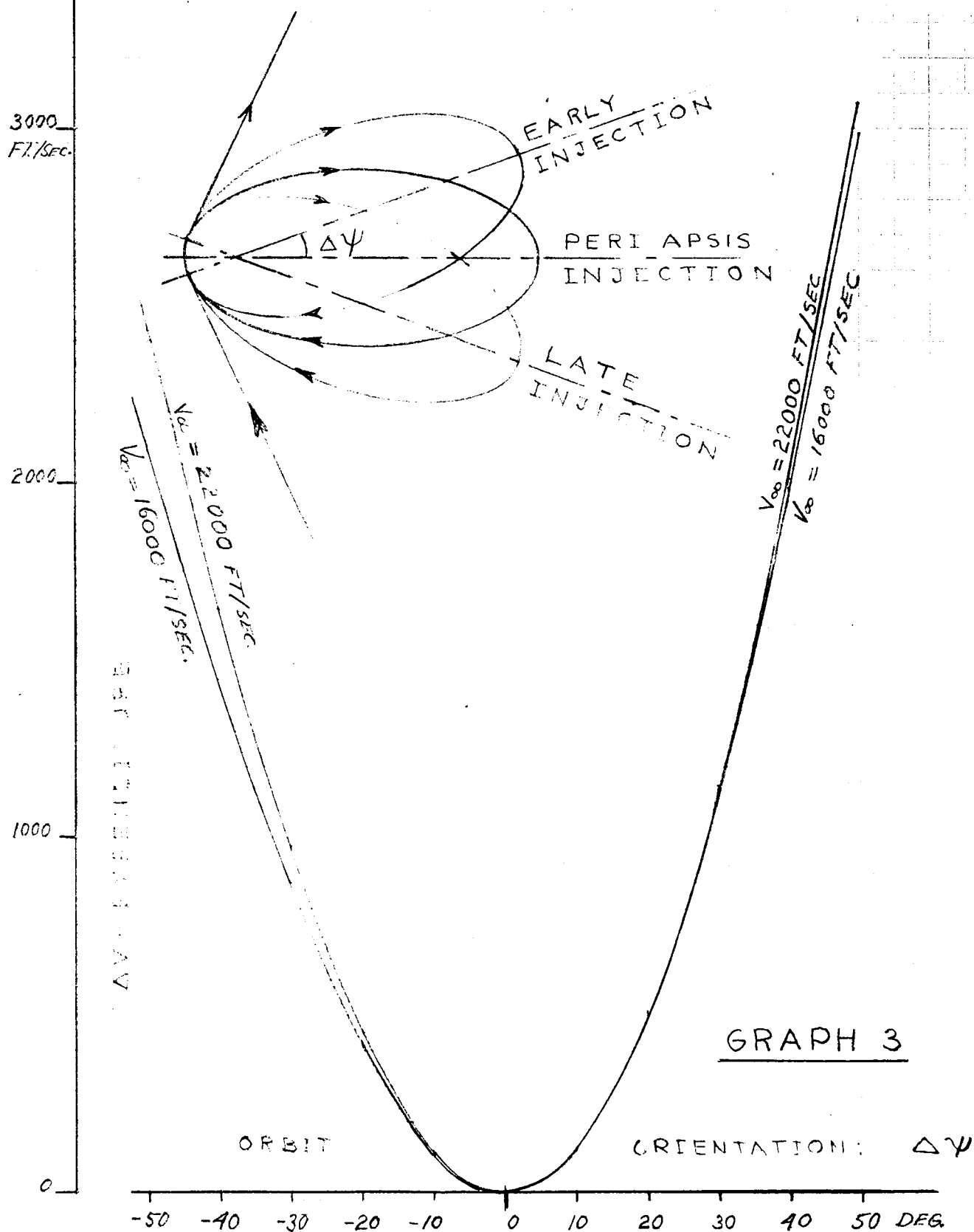








ΔV - EXPENDITURE FOR OFF PERI APSIS
INJECTION VS. ORBIT ORIENTATION $\Delta\psi$.



GRAPH 4 DEPLOYMENT FROM A CIRCULAR ORBIT. DEPLOYMENT ΔV VS. DEPLOYMENT ANOMALY FOR 0 DEG. ENTRY ANOMALY.

2500
Ft/sec

DEPLOYMENT ΔV

2000

1500

ANOMALY OF DEPLOYMENT POINT

1000

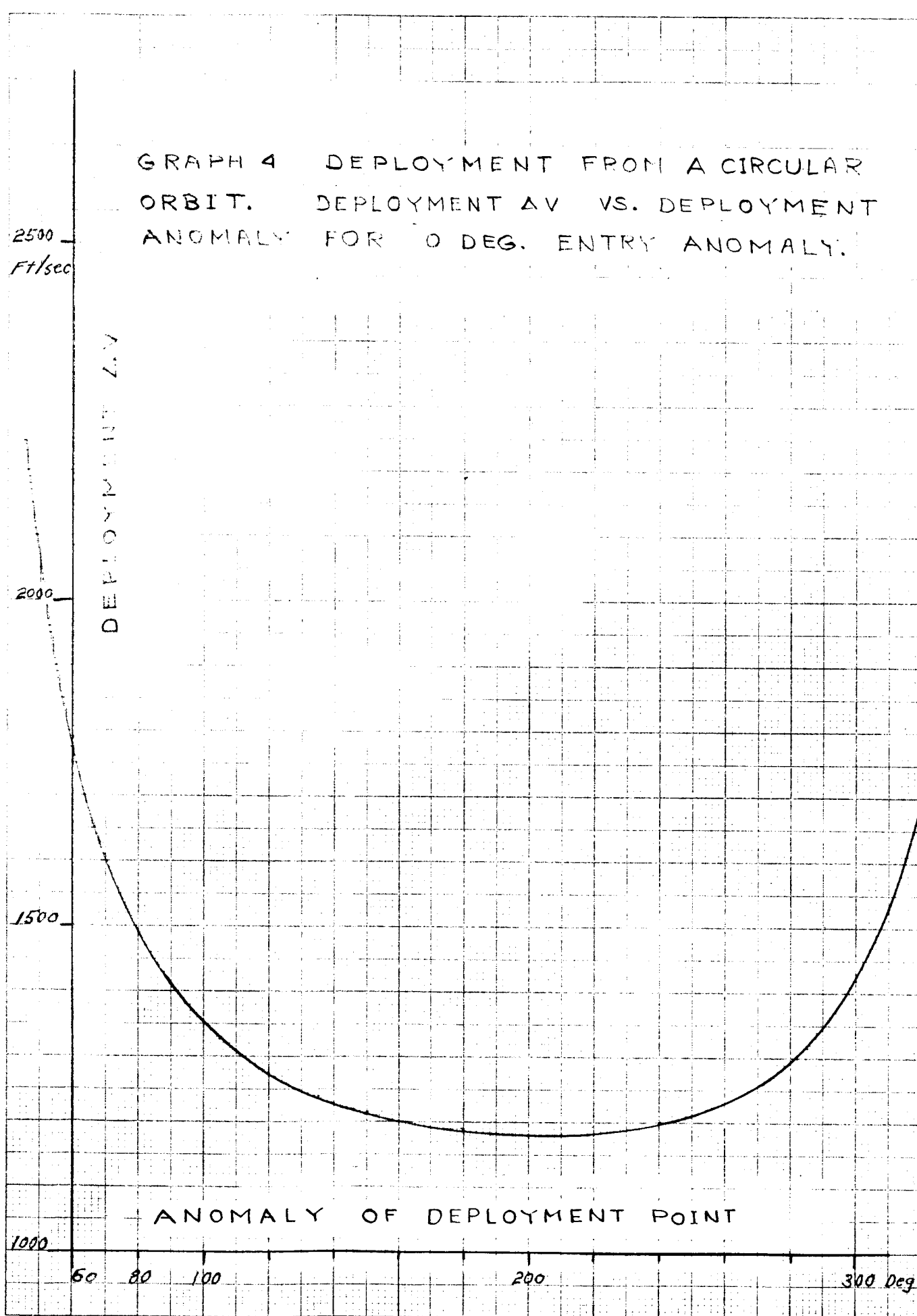
60

80

100

200

300 Deg



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REFERENCES

1. Schoch, J. J., "1975 Mars Flyby Mission - Trajectories of Probes from Manned Spacecraft", Case 103-2, July 6, 1966.
2. Schoch, J. J., "1975 Venus Lightside Flyby Trajectory of Spacecraft and Probes", Case 233, June 7, 1967.
3. London, H. S., "Escape from Elliptical Parking Orbits" AAS Preprint 66-127, July, 1966.

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